Find the general solution of the differential equation $yy'' = (y-1)(y')^2$. LET $u = y' = \frac{dy}{dx}$ **SCORE: _____ / 8 PTS**

$$y'' = \frac{d^2y}{dx^2} = u\frac{du}{dy}$$

2 $yu\frac{du}{dy} = (y-1)u^2$

$\frac{1}{u}du = \frac{y-1}{y}dy = (1-\frac{1}{y})dy$

ALL ITEMS WORTH 1 POINT UNLESS OTHERWISE NOTED

$\ln|u| = y - \ln|y| + C$

$\frac{dy}{dx} = u = Cy^{-1}e^y$

$Cye^{-y}dy = dx$

$C(-ye^{-y} - e^{-y}) + K = x$

$x = Ce^{-y}(y+1) + K$

y	e^{-y}
1	$-e^{-y}$
0	e^{-y}

Using elimination **as shown in lecture**, solve the system of differential equations **SCORE: _____ / 22 PTS**

$$3x' + 4y' + x + y = 5t + 1 \qquad ① \quad (3D+1)[x] + (4D+1)[y] = 5t+1$$

$$2x' + 3y' - x - y = 3t \qquad ② \quad (2D-1)[x] + (3D-1)[y] = 3t$$

APPLY $(3D-1)$ TO ① $\quad (3D-1)(3D+1)[x] + (3D-1)(4D+1)[y] = 15-(5t+1)$

$-(4D+1)$ TO ② $\quad -(4D+1)(2D-1)[x] - (4D+1)(3D-1)[y] = -(12+3t)$

ADD $\quad (D^2+2D)[x] = 2-8t$

$r = 0, -2 \longrightarrow x_h = c_1 + c_2 e^{-2t}$

$x_p = (At+B)t = At^2 + Bt$

$x_p' = 2At + B$

$x_p'' = 2A$

$+2x_p' \quad + \; 4At + 2B$

$= 4At + 2A + 2B$

$4A = -8 \rightarrow A = -2$

$2A + 2B = 2 \rightarrow B = 1 - A$
$= 3$

$x = -2t^2 + 3t + c_1 + c_2 e^{-2t}$

ALL ITEMS WORTH
① POINT
UNLESS OTHERWISE NOTED

APPLY $-(2D-1)$ TO ① $\quad -(2D-1)(3D+1)[x] - (2D-1)(4D+1)[y] = -(10-(5t+1))$

$(3D+1)$ TO ② $\quad (3D+1)(2D-1)[x] + (3D+1)(3D-1)[y] = 9 + 3t$

ADD $\quad (D^2+2D)[y] = 8t$

$y_h = k_1 + k_2 e^{-3t}$

$y_p = Ct^2 + Dt$

$4C = 8 \rightarrow C = 2$

$2C + 2D = 0 \rightarrow D = -C$
$= -2$

$y = 2t^2 - 2t + k_1 + k_2 e^{-2t}$

SUBSTITUTING INTO ①

$3x' = 3(-4t + 3 - 2c_2 e^{-2t})$

$+4y' \quad +4(4t - 2 - 2k_2 e^{-2t})$

$+ \; x \quad -2t^2 + 3t + c_1 + c_2 e^{-2t}$

$+ \; y \quad 2t^2 - 2t + k_1 + k_2 e^{-2t}$

② $= 5t + (1 + c_1 + k_1) + (-5c_2 - 7k_2) e^{-2t}$

$1 + c_1 + k_1 = 1 \qquad -5c_2 - 7k_2 = 0$

$k_1 = -c_1 \qquad k_2 = -\frac{5}{7} c_2$

$x = -2t^2 + 3t + c_1 + c_2 e^{-2t}$

$y = 2t^2 - 2t - c_1 - \frac{5}{7} c_2 e^{-2t}$